# Game Theory, Spring 2024

# Problem Set # 6

Daniil Larionov

Due May 29 at 5:15 PM

**Exercise 1**

Consider the following infinitely repeated prisoner's dilemma:

| | $c$ | $d$ |
|---|---|---|
| $c$ | $5, 5$ | $1, 6$ |
| $d$ | $6, 1$ | $2, 2$ |

1. Consider the following strategy (*tit-for-tat*): play $c$ in the initial period; in any other period, play the action your opponent played in the previous period.
   (a) Write down the automaton representation of the strategy profile (*tit-for-tat, tit-for-tat*) and provide its graphical illustration.
   (b) Use the one-shot deviation principle to determine the range of discount factors (if any) for which (*tit-for-tat, tit-for-tat*) is a subgame-perfect Nash equilibrium of this prisoner's dilemma.
2. Consider the following strategy $\sigma_i$: play $c$ in the initial period; in any other period, play $c$ if $(c, c)$ or $(d, d)$ was played in the previous period and play $d$ if $(c, d)$ or $(d, c)$ was played in the previous period.
   (a) Write down the automaton representation of the strategy profile $(\sigma_1, \sigma_2)$ and provide its graphical illustration.
   (b) Use the one-shot deviation principle to determine the range of discount factors (if any) for which $(\sigma_1, \sigma_2)$ is a subgame-perfect Nash equilibrium of this prisoner's dilemma.

(c) Let $\delta = \frac{2}{3}$, check whether $\{(5,5);(4,4)\}$ is a self-generating set.

3. Consider the following strategy $\sigma_i$: there are three phases: *the regular phase*, *the 1-phase*, and *the 2-phase*. In the regular phase, player $i$ plays $c$. If $(c,c)$ or $(d,d)$ is played in the regular phase, the play stays in the regular phase. If $(d,c)$ is played in the regular phase, then the play moves to the 1-phase. If $(c,d)$ is played in the regular phase, then the play moves to the 2-phase. In the $i$-phase, player $i$ plays $c$, player $-i$ plays $d$. If, in the $i$-phase, player $i$ plays $c$ and player $-i$ plays $d$ or both players play $c$, the play moves back to the regular phase; otherwise the play remains in the $i$-phase.

   (a) Write down the automaton representation of the strategy profile $(\sigma_1, \sigma_2)$ and provide its graphical illustration.

   (b) Use the one-shot deviation principle to determine the range of discount factors (if any) for which $(\sigma_1, \sigma_2)$ is a subgame-perfect Nash equilibrium of this prisoner's dilemma.

   (c) Let $\delta = \frac{1}{2}$, check whether $\{(5,5);(5.5,3);(3,5.5)\}$ is a self-generating set.

**Exercise 2**

Consider the following infinitely repeated game:

| | $c$ | $k$ | $d$ |
|---|---|---|---|
| $c$ | $4,4$ | $3,0$ | $1,0$ |
| $k$ | $0,3$ | $2,2$ | $1,0$ |
| $d$ | $0,1$ | $0,1$ | $0,0$ |

Determine the range of discount factors $\delta$ (if any), for which $\{(2,2);(2\delta,2\delta);(2\delta^2,2\delta^2)\}$ is a self-generating set.